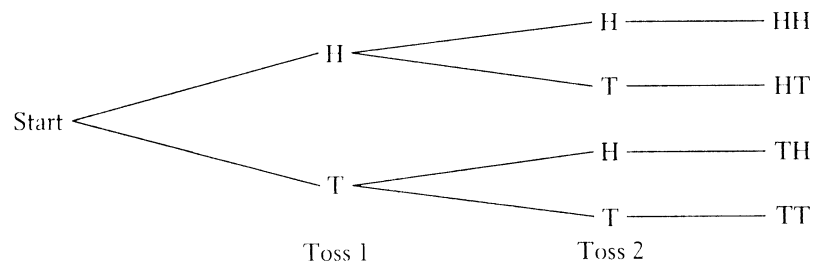


Probability: The Study of Randomness

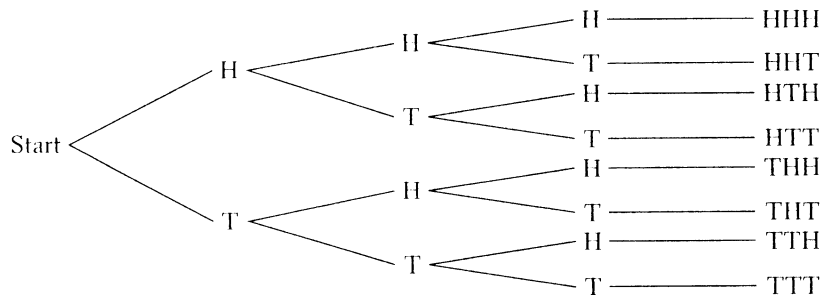
- 6.1 Long trials of this experiment often approach 40% heads. One theory attributes this surprising result to a “bottle-cap effect” due to an unequal rim on the penny. We don’t know. But a teaching assistant claims to have spent a profitable evening at a party betting on spinning coins after learning of the effect.
- 6.2 The theoretical probabilities are, in order: $1/16$, $4/16 = 1/4$, $6/16 = 3/8$, $4/16 = 1/4$, $1/16$.
- 6.3 (b) In our simulation, Shaq hit 52% of his shots. (c) The longest sequence of misses in our run was 6 and the longest sequence of hits was 9. Of course, results will vary.
- 6.4 (a) 0. (b) 1. (c) 0.01. (d) 0.6 (or 0.99, but “more often than not” is a rather weak description of an event with probability 0.99!)
- 6.5 There are 21 0s among the first 200 digits; the proportion is $\frac{21}{200} = 0.105$.
- 6.6 (a) We expect probability $1/2$ (for the first flip, or for *any* flip of the coin). (b) The theoretical probability that the first head appears on an odd-numbered toss of a fair coin is $\frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \dots = \frac{2}{3}$. Most answers should be between about 0.47 and 0.87.
- 6.7 Obviously, results will vary with the type of thumbtack used. If you try this experiment, note that although it is commonly done when flipping coins, we do not recommend throwing the tack in the air, catching it, and slapping it down on the back of your other hand.
- 6.8 In the long run, of a large number of hands of five cards, about 2% (one out of 50) will contain a three of a kind. (Note: This probability is actually $\frac{88}{4165} \doteq 0.02113$.)
- 6.9 The study looked at regular season games, which included games against poorer teams, and it is reasonable to believe that the 63% figure is inflated because of these weaker opponents. In the World Series, the two teams will (presumably) be nearly the best, and home game wins will not be so easy.
- 6.10 (a) With $n = 20$, nearly all answers will be 0.40 or greater. With $n = 80$, nearly all answers will be between 0.58 and 0.88. With $n = 320$, nearly all answers will be between 0.66 and 0.80.
- 6.11 (a) $S = \{\text{germinates, fails to grow}\}$. (b) If measured in weeks, for example, $S = \{0, 1, 2, \dots\}$. (c) $S = \{A, B, C, D, F\}$. (d) Using Y for “yes (shot made)” and N for “no (shot missed),” $S = \{YYYY, NNNN, YYYN, NNNY, YNYN, NNYN, YNNY, NYNN, NYYY, YNNN, YYNN, NNNY, YNNY, YNNY, YNNY\}$. (There are 16 items in the sample space.) (e) $S = \{0, 1, 2, 3, 4\}$.
- 6.12 (a) $S = \{\text{all numbers between 0 and 24}\}$. (b) $S = \{0, 1, 2, \dots, 11000\}$. (c) $S = \{0, 1, 2, \dots, 12\}$. (d) $S = \{\text{all numbers greater than or equal to 0}\}$, or $S = \{0, 0.01, 0.02, 0.03, \dots\}$. (e) $S = \{\text{all positive and negative numbers}\}$. Note that the rats can lose weight.

6.13 $S = \{\text{all numbers between } \underline{\hspace{1cm}} \text{ and } \underline{\hspace{1cm}}\}$. The numbers in the blanks may vary. Table 1.10 has values from 86 to 195 cal; the range of values in S should include *at least* those numbers. Some students may play it safe and say “all numbers greater than 0.”

6.14 If two coins are tossed, then by the multiplication principle, there are $(2)(2) = 4$ possible outcomes. The outcomes are illustrated in the following tree diagram:



The sample space is $\{HH, HT, TH, TT\}$. (b) If three coins are tossed, then there are $(2)(2)(2) = 8$ possible outcomes. The outcomes are illustrated in the following tree diagram:



The sample space is $\{HHHH, HHHT, HHTH, HHTT, THHH, THT, TTH, TTT\}$. (c) If four coins are tossed, then there are $(2)(2)(2)(2) = 16$ possible outcomes, each of which consists of a string of four letters that may be H's or T's. The sample space is $\{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THTH, TTHH, THHT, HTTT, THTT, TTHT, TTTT\}$.

6.15 (a) $10 \times 10 \times 10 \times 10 = 10^4 = 10,000$. (b) $10 \times 9 \times 8 \times 7 = 5,040$. (c) There are 10,000 four-digit tags, 1,000 three-digit tags, 100 two-digit tags, and 10 one-digit tags, for a total of 11,110 license tags.

6.16 (a) An outcome of this experiment consists of a string of 3 digits, each of which can be 1, 2, or 3. By the multiplication principle, the number of possible outcomes is $(3)(3)(3) = 27$. (b) The sample space is $\{111, 112, 113, 121, 122, 123, 131, 132, 133, 211, 212, 213, 221, 222, 223, 231, 232, 233, 311, 312, 313, 321, 322, 323, 331, 332, 333\}$.

6.17 (a) Number of ways	Sum	Outcomes
1	2	(1, 1)
2	3	(1, 2) (2, 1)
3	4	(1, 3) (2, 2) (3, 1)
4	5	(1, 4) (2, 3) (3, 2) (4, 1)
5	6	(1, 5) (2, 4) (3, 3) (4, 2) (5, 1)
6	7	(1, 6) (2, 5) (3, 4) (4, 3) (5, 2) (6, 1)
5	8	(2, 6) (3, 5) (4, 4) (5, 3) (6, 2)
4	9	(3, 6) (4, 5) (5, 4) (6, 3)
3	10	(4, 6) (5, 5) (6, 4)
2	11	(5, 6) (6, 5)
1	12	(6, 6)

(b) 18.

(c) There are 4 ways to get a sum of 5 and 5 ways to get a sum of 8.

(d) Answers will vary but might include:

- The “number of ways” increases until “sum = 7” and then decreases.
- The “number of ways” is symmetrical about “sum = 7.”
- Odd sums occur an even number of ways and even sums occur an odd number of ways.

6.18 (a) 26. (b) 13. (c) 1. (d) 16. (e) 3.

6.19 (a) The given probabilities have sum 0.96, so $P(\text{type AB}) = 0.04$. (b) $P(\text{type O or B}) = 0.49 + 0.20 = 0.69$.

6.20 (a) The sum of the given probabilities is 0.9, so $P(\text{blue}) = 0.1$. (b) The sum of the given probabilities is 0.7, so $P(\text{blue}) = 0.3$. (c) $P(\text{plain M\&M is red, yellow, or orange}) = 0.2 + 0.2 + 0.1 = 0.5$. $P(\text{peanut M\&M is red, yellow, or orange}) = 0.1 + 0.2 + 0.1 = 0.4$.

6.21 $P(\text{either CV disease or cancer}) = 0.45 + 0.22 = 0.67$; $P(\text{other cause}) = 1 - 0.67 = 0.33$.

6.22 (a) Since the three probabilities must add to 1 (assuming that there were no “no opinion” responses), this probability must be $1 - (0.12 + 0.61) = 0.27$. (b) $0.12 + 0.61 = 0.73$.

6.23 (a) The sum is 1, as we expect since all possible outcomes are listed. (b) $1 - 0.41 = 0.59$. (c) $0.41 + 0.23 = 0.64$. (d) $(0.41)(0.41) = 0.1681$.

6.24 There are 19 outcomes where at least one digit occurs in the correct position: 111, 112, 113, 121, 122, 123, 131, 132, 133, 213, 221, 222, 223, 233, 313, 321, 322, 323, 333. The theoretical probability of at least one digit occurring in the correct position is therefore $19/27 = .7037$.

6.25 (a) Let x = number of spots. Then $P(x = 1) = P(x = 2) = P(x = 3) = P(x = 4) = 0.25$. Since all 4 faces have the same shape and the same area, it is reasonable to assume that the probability of a face being down is the same as for any other face. Since the sum of the probabilities must be one, the probability of each should be 0.25.

(b) Outcomes (1,1) (1,2) (1,3) (1,4) (2,1) (2,2) (2,3) (2,4) (3,1) (3,2) (3,3) (3,4) (4,1) (4,2) (4,3) (4,4)
The probability of any pair is $1/16 = 0.0625$.

$P(\text{Sum} = 5) = P(1,4) + P(2,3) + P(3,2) + P(4,1) = (0.0625)(4) = 0.25$.

6.26 (a) $P(D) = P(1, 2, \text{ or } 3) = .301 + .176 + .125 = .602$.

(b) $P(B \cup D) = P(B) + P(D) = .602 + .222 = .824$.

(c) $P(D^c) = 1 - P(D) = 1 - .602 = .398$.

(d) $P(C \cap D) = P(1 \text{ or } 3) = .301 + .125 = .426$.

(e) $P(B \cap C) = P(7 \text{ or } 9) = .058 + .046 = .104$.

6.27 Fight one big battle: His probability of winning is 0.6, compared to $0.8^3 = 0.512$. (Or he could choose to try for a negotiated peace.)

6.28 $(1 - 0.05)^{12} = (0.95)^{12} = 0.5404$.

6.29 No: It is unlikely that these events are independent. In particular, it is reasonable to expect that college graduates are less likely to be laborers or operators.

6.30 (a) $P(A) = \frac{38,225}{166,438} = 0.230$ since there are 38,225 (thousand) people who have completed 4+ years of college out of 166,438 (thousand). (b) $P(B) = \frac{52,022}{166,438} = 0.313$. (c) $P(A \text{ and } B) = \frac{8,005}{166,438} = 0.048$; A and B are not independent since $P(A \text{ and } B) \neq P(A)P(B)$.

6.31 An individual light remains lit for 3 years with probability $1 - 0.02$; the whole string remains lit with probability $(1 - 0.02)^{20} = (0.98)^{20} = 0.6676$.

6.32 $P(\text{neither test is positive}) = (1 - 0.9)(1 - 0.8) = (0.1)(0.2) = 0.02$.

6.33 (a) $P(\text{one call does not reach a person}) = 0.8$. Thus, $P(\text{none of the 5 calls reaches a person}) = (0.8)^5 = 0.32768$.

(b) $P(\text{one call to NYC does not reach a person}) = 0.92$. Thus, $P(\text{none of the 5 calls to NYC reach a person}) = (0.92)^5 = 0.6591$.

6.34 Model 1: Legitimate. Model 2: Legitimate. Model 3: Probabilities have sum $\frac{6}{7}$. Model 4: Probabilities cannot be negative.

6.35 (a) Legitimate. (b) Not legitimate, because probabilities sum to more than 1. (c) Not legitimate, because probabilities sum to less than 1.

6.36 (a) Sum of given probabilities = .705, so $P(\text{car has some other color}) = 1 - .705 = .295$.

(b) $P(\text{silver or white}) = P(\text{silver}) + P(\text{white}) = .176 + .172 = .348$.

(c) Assuming that the vehicle choices are independent, $P(\text{both silver or white}) = (.348)^2 = .121$.

6.37 (a) The sum of all 8 probabilities equals 1 and all probabilities satisfy $0 \leq p \leq 1$.

(b) $P(A) = 0.000 + 0.003 + 0.060 + 0.062 = 0.125$.

(c) The chosen person is not white.

$P(B^c) = 1 - P(B) = 1 - (0.060 + 0.691) = 1 - 0.751 = 0.249$.

(d) $P(A^c \cap B) = 0.691$.

6.38 A, B are not independent because $P(A \text{ and } B) = 0.06$, but $P(A) \times P(B) = (.125)(.751) = 0.093875$. For the events to be independent, these two probabilities must be equal.

6.39 (a) $P(\text{undergraduate and score} \geq 600) = (0.40)(0.50) = 0.20$.

$P(\text{graduate and score} \geq 600) = (0.60)(0.70) = 0.42$.

(b) $P(\text{score} \geq 600) = P(\text{UG and score} \geq 600) + P(\text{G and score} \geq 600) = 0.20 + 0.42 = 0.62$.

6.40 (a) $(0.65)^3 = 0.2746$ (under the random walk theory). (b) 0.35 (since performance in separate years is independent). (c) $(0.65)^2 + (0.35)^2 = 0.545$.

6.41 (a) $P(\text{under 65}) = 0.321 + 0.124 = 0.445$. $P(65 \text{ or older}) = 1 - 0.445 = 0.555$. (b) $P(\text{tests done}) = 0.321 + 0.365 = 0.686$. $P(\text{tests not done}) = 1 - 0.686 = 0.314$. (c) $P(A \text{ and } B) = 0.365$; $P(A)P(B) = (0.555)(0.686) = 0.3807$. A and B are not independent; tests were done less frequently on older patients than if these events were independent.

6.42 (a) $1/38$. (b) Since 18 slots are red, the probability of a red is $P(\text{red}) = \frac{18}{38} = 0.474$. (c) There are 12 winning slots, so $P(\text{win a column bet}) = \frac{12}{38} = 0.316$.

6.43 Look at the first five rolls in each sequence. All have one G and four R's, so those probabilities are the same. In the first sequence, you win regardless of the sixth roll; for the second, you win if the sixth roll is G, and for the third sequence, you win if it is R. The respective probabilities are $(\frac{2}{6})^4 \cdot (\frac{4}{6}) = \frac{2^7}{3^4 \cdot 3} = 0.00823$, $(\frac{2}{6})^4 \cdot (\frac{4}{6})^2 = \frac{4}{729} = 0.00549$, and $(\frac{2}{6})^5 \cdot (\frac{4}{6}) = \frac{2^7}{729} = 0.00274$.

6.44 $P(\text{first child is albino}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. $P(\text{both of two children are albino}) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$. $P(\text{neither is albino}) = (1 - \frac{1}{4})^2 = \frac{9}{16}$.

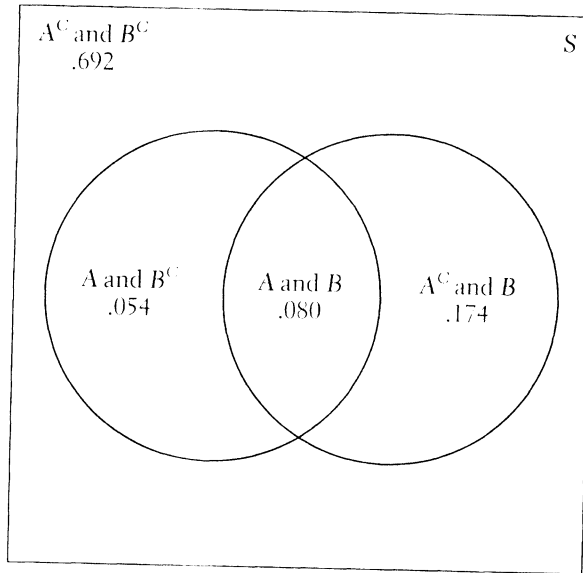
6.45 (a) If A, B are independent, then $P(A \text{ and } B) = P(A) \cdot P(B)$. Since A and B are nonempty, then we have $P(A) > 0$, $P(B) > 0$ and $P(A) \cdot P(B) > 0$. Therefore, $P(A \text{ and } B) > 0$. So A and B cannot be empty.

(b) If A and B are disjoint, then $P(A \text{ and } B) = 0$. But this cannot be true if A and B are independent by part (a). So A and B cannot be independent.

(c) Example: A bag contains 3 red balls and 2 green balls. A ball is drawn from the bag, its color is noted, and the ball is set aside. Then a second ball is drawn and its color is noted. Let event A be the event that the first ball is red. Let event B be the event that the second ball is red. Events A and B are not disjoint because both balls can be red. However, events A and B are not independent because whether the first ball is red or not, alters the probability of the second ball being red.

6.46 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.125 + 0.237 - 0.077 = 0.285.$

6.47



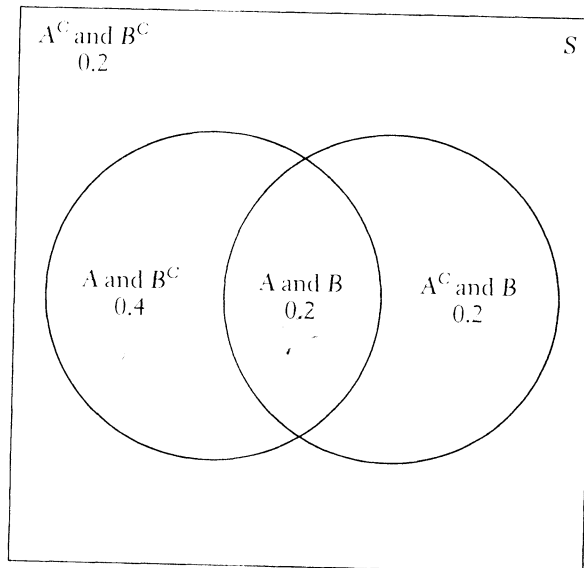
(a) $\{A \text{ and } B\}$ represents both prosperous and educated. $P(A \text{ and } B) = 0.080$. (b) $\{A \text{ and } B^c\}$ represents prosperous but not educated. $P(A \text{ and } B^c) = P(A) - P(A \text{ and } B) = .134 - .080 = .054$. (c) $\{A^c \text{ and } B\}$ represents not prosperous but educated. $P(A^c \text{ and } B) = P(B) - P(A \text{ and } B) = .254 - .080 = .174$. (d) $\{A^c \text{ and } B^c\}$ represents neither prosperous nor educated. $P(A^c \text{ and } B^c) = 1 - (.054 + .080 + .174) = 1 - .308 = .692$.

6.48 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.6 + 0.4 - 0.2 = 0.8.$

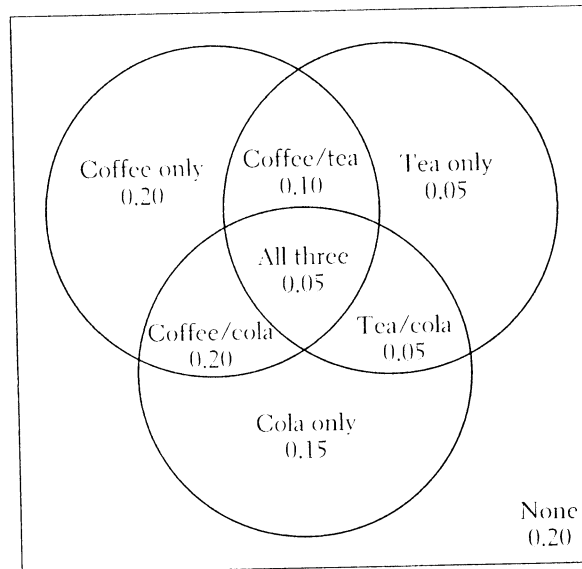
6.49 $P(A) \cdot P(B) = (0.6)(0.5) = 0.30$

Since this equals the stated probability for $P(A \text{ and } B)$, events A and B are independent.

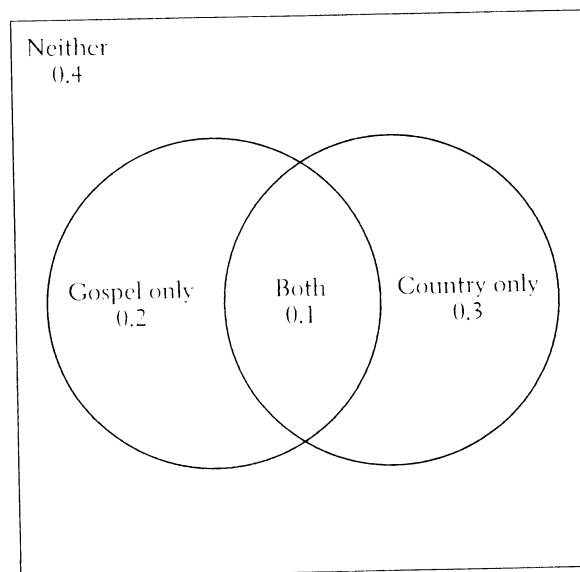
6.50 (a) This event is $\{A \text{ and } B\}$; $P(A \text{ and } B) = 0.2$ (given). (b) This is $\{A \text{ and } B^c\}$; $P(A \text{ and } B^c) = P(A) - P(A \text{ and } B) = 0.4$. (c) This is $\{A^c \text{ and } B\}$; $P(A^c \text{ and } B) = P(B) - P(A \text{ and } B) = 0.2$. (d) This is $\{A^c \text{ and } B^c\}$; $P(A^c \text{ and } B^c) = 0.2$ (so that the probabilities add to 1).



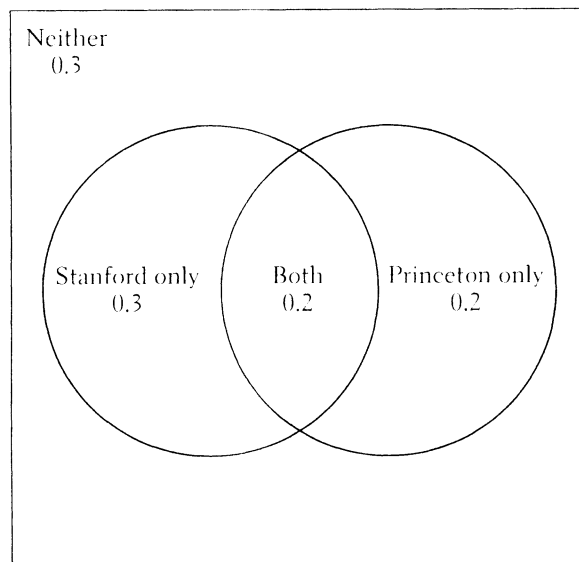
6.51 In constructing the Venn diagram, start with the numbers given for “only tea” and “all three,” then determine other values. For example, $P(\text{coffee and cola, but not tea}) = P(\text{coffee and cola}) - P(\text{all three})$. (a) 15% drink only cola. (b) 20% drink none of these.



6.52 (a) Below. (b) $P(\text{country but not Gospel}) = P(C) - P(C \text{ and } G) = 0.4 - 0.1 = 0.3$. (c) $P(\text{neither}) = 1 - P(C \text{ or } G) = 1 - (0.4 + 0.3 - 0.1) = 0.4$.



6.53 (a) On the facing page. (b) $P(\text{neither admits Ramon}) = 1 - P(P \text{ or } S) = 1 - (0.4 + 0.5 - 0.2) = 0.3$. (c) $P(S \text{ and not } P) = P(S) - P(P \text{ and } S) = 0.3$.



6.54 (a) $18669/103870 = .18$.

(b) $8270/18669 = .443$.

(c) $8270/103870 = .08$.

6.55 (a) $7842/59920 = 0.13087$.

(b) Married, age 18 to 29.

(c) 0.13087 is the proportion of women who are age 18 to 29 among those women who are married.

6.56 $P(A \text{ and } B) = P(A) P(B | A) = (0.46)(0.32) = 0.1472$.

6.57 If $F = \{\text{dollar falls}\}$ and $R = \{\text{renegotiation demanded}\}$, then $P(F \text{ and } R) = P(F) P(R | F) = (0.4)(0.8) = 0.32$.

6.58 (a) & (b) These probabilities are below. (c) The product of these conditional probabilities gives the probability of a flush in spades by the extended multiplication rule: We must draw a spade, and then another, and then a third, a fourth, and a fifth. The product of these probabilities is about 0.0004952. (d) Since there are four possible suits in which to have a flush, the probability of a flush is four times that found in (c), or about 0.001981.

$$P(\text{1st card } \spadesuit) = \frac{13}{52} = \frac{1}{4} = 0.25$$

$$P(\text{2nd card } \spadesuit | 1 \spadesuit \text{ picked}) = \frac{12}{51} = \frac{4}{17} \doteq 0.2353$$

$$P(\text{3rd card } \spadesuit | 2 \spadesuit \text{ s picked}) = \frac{11}{50} \doteq 0.22$$

$$P(\text{4th card } \spadesuit | 3 \spadesuit \text{ s picked}) = \frac{10}{49} \doteq 0.2041$$

$$P(\text{5th card } \spadesuit | 4 \spadesuit \text{ s picked}) = \frac{9}{48} = \frac{3}{16} = 0.1875$$

6.59 First, concentrate on (say) spades. The probability that the first card dealt is one of those five cards ($A\spadesuit, K\spadesuit, Q\spadesuit, J\spadesuit, \text{ or } 10\spadesuit$) is $5/52$. The conditional probability that the second is one of those cards, given that the first was, is $4/51$. Continuing like this, we get $3/50$, $2/49$, and finally $1/48$; the product of these five probabilities gives $P(\text{royal flush in spades}) \doteq 0.00000038477$. Multiplying by four gives $P(\text{royal flush}) \doteq 0.000001539$.

6.60 (a) $P(\text{income} \geq \$50,000) = 0.20 + 0.05 = 0.25$.

$$(b) P(\text{income} \geq \$100,000 \mid \text{income} \geq \$50,000) = 0.05/0.25 = 0.2.$$

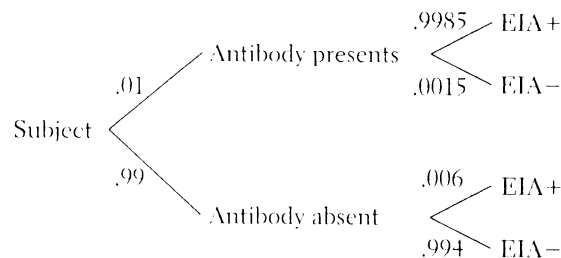
6.61 Let $G = \{\text{student likes Gospel}\}$ and $C = \{\text{student likes country}\}$. See the Venn diagram in the solution to Exercise 6.52. (a) $P(G \mid C) = P(G \text{ and } C)/P(C) = 0.1/0.4 = 0.25$. (b) $P(G \mid \text{not } C) = P(G \text{ and not } C)/P(\text{not } C) = 0.2/0.6 = \frac{1}{3} \doteq 0.33$.

6.62 (a) $P(\text{at least } \$100,000) = 9,534,653/127,075,145 = .075$; $P(\text{at least } \$1 \text{ million}) = 205,124/127,075,145 = .002$.

$$(b) P(\text{at least } \$1 \text{ million} \mid \text{at least } \$100,000) = .002/.075 = .027.$$

6.63 Let $I =$ the event that the operation results in infection, $F =$ the event that the operation fails. We seek $P(I^c \text{ and } F^c)$. We are given that $P(I) = .03$, $P(F) = .14$, and $P(I \text{ and } F) = .01$. Then $P(I^c \text{ and } F^c) = 1 - P(I \text{ or } F) = 1 - (.03 + .14 - .01) = .84$.

6.64 (a)



$$(b) P(\text{test pos}) = P(\text{antibody and test pos}) + P(\text{no antibody and test pos.}) = (.01)(.9985) + (.99)(.006) = .016.$$

$$(c) P(\text{antibody} \mid \text{test pos}) = P(\text{antibody and test pos})/P(\text{test pos}) = (.01)(.9985)/.016 = .624$$

$$6.65 (a) P(\text{antibody} \mid \text{test pos}) = \frac{0.0009985}{0.0009985 + 0.005994} = 0.1428.$$

$$(b) P(\text{antibody} \mid \text{test pos}) = \frac{0.00985}{0.00985 + 0.0054} = 0.9487.$$

(c) A positive result does not always indicate that the antibody is present. How common a factor is in the population can impact the test probabilities.

6.66 (a) $P(\text{chemistry}) = 110/445 = .247$. 24.7% of all laureates won prizes in chemistry.

(b) $P(\text{US}) = 198/445 = .445$. 44.5% of all laureates did research in the United States.

(c) $P(\text{US} \mid \text{phys-med}) = 82/144 = .569$. 56.9% of all physiology/medicine laureates did research in the United States.

(d) $P(\text{phys-med} \mid \text{US}) = 82/198 = .414$. 41.4% of all laureates from the United States won prizes in physiology/medicine.

6.67 (a) $\frac{856}{1,626} = 0.5264$. (b) $\frac{31}{74} = 0.4054$. (c) No: If they were independent, the answers to (a) and (b) would be the same.

6.68 (a) $P(\text{jack}) = 1/13$.

$$(b) P(5 \text{ on second} \mid \text{jack on first}) = 1/12.$$

$$(c) P(\text{jack on first and 5 on second}) = P(\text{jack on first}) \times P(5 \text{ on second} \mid \text{jack on first}) = (1/13) \times (1/12) = 1/126.$$

$$(d) P(\text{both cards greater than 5}) = P(\text{first card greater than 5}) \times P(\text{second card greater than 5} \mid \text{first card greater than 5}) = (8/13) \times (7/12) = 56/126 = 4/9.$$

6.69 (a) $\frac{570}{1626} \approx 0.4736$. (b) $\frac{529}{770} = 0.6870$. (d) Using multiplication rule: $P(\text{male and bachelor's degree}) = P(\text{male})P(\text{bachelor's degree} | \text{male}) = (0.4736)(0.6870) = 0.3254$. (Answers will vary with how much previous answers had been rounded.) Directly: $\frac{529}{1626} \approx 0.3253$. [Note that the difference between these answers is inconsequential, since the numbers in the table are rounded.]

6.70 (a) $P(C) = 0.20$, $P(A) = 0.10$, $P(A | C) = 0.05$.

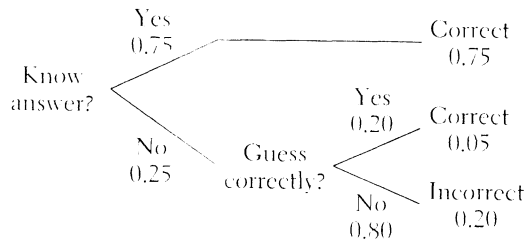
(b) $P(A \text{ and } C) = P(C) \times P(A | C) = (0.20)(0.05) = 0.01$.

6.71 Percent of "A" students involved in an accident is

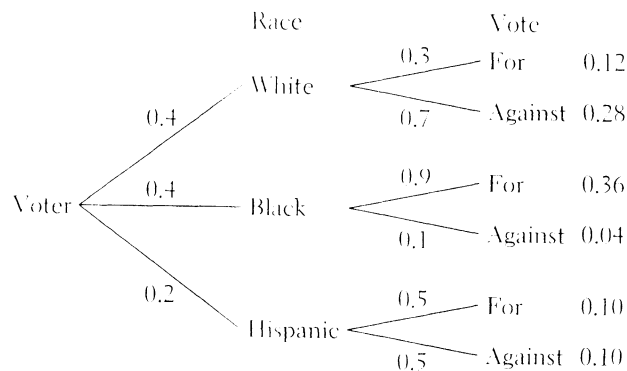
$$P(C | A) = \frac{P(C \text{ and } A)}{P(A)} = \frac{0.01}{0.10} = 0.10.$$

6.72 If $F = \{\text{dollar falls}\}$ and $R = \{\text{renegotiation demanded}\}$, then $P(R) = P(F \text{ and } R) + P(F^c \text{ and } R) = 0.32 + P(F^c)P(R | F^c) = 0.32 + (0.6)(0.2) = 0.44$.

6.73 $P(\text{correct}) = P(\text{knows answer}) + P(\text{doesn't know, but guesses correctly}) = 0.75 + (0.25)(0.20) = 0.8$.



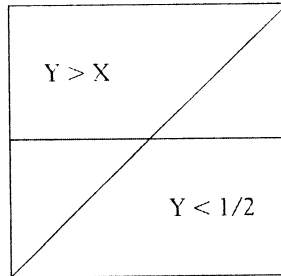
6.74 Tree diagram below. The black candidate expects to get $12\% + 36\% + 10\% = 58\%$ of the vote.



6.75 $P(\text{knows the answer} | \text{gives the correct answer}) = \frac{0.75}{0.80} = \frac{15}{16} = 0.9375$.

6.76 The event $\{Y < 1/2\}$ is the bottom half of the square, while $\{Y > X\}$ is the upper left triangle of the square. They overlap in a triangle with area $1/8$, so

$$P(Y < \frac{1}{2} | Y > X) = \frac{P(Y < \frac{1}{2} \text{ and } Y > X)}{P(Y > X)} = \frac{1/8}{1/2} = \frac{1}{4}$$



6.77 (a) $P(\text{switch bad}) = \frac{1000}{10,000} = 0.1$; $P(\text{switch OK}) = 1 - P(\text{switch bad}) = 0.9$. (b) Of the 9999 remaining switches, 999 are bad. $P(\text{second bad} | \text{first bad}) = \frac{999}{9999} \doteq 0.09991$. (c) Of the 9999 remaining switches, 1000 are bad. $P(\text{second bad} | \text{first good}) = \frac{1000}{9999} \doteq 0.10001$.

6.78 (a) There are 10 pairs. Just using initials: $\{(A,D), (A,J), (A,S), (A,R), (D,J), (D,S), (D,R), (J,S), (J,R), (S,R)\}$. (b) Each has probability $1/10 = 10\%$. (c) Julie is chosen in 4 of the 10 possible outcomes: $4/10 = 40\%$. (d) There are 3 pairs with neither Sam nor Roberto, so the probability is $3/10$.

6.79 (a) $P(\text{Type AB}) = 1 - (0.45 + 0.40 + 0.11) = 0.04$.

(b) $P(\text{Type B or Type O}) = 0.11 + 0.45 = 0.56$.

(c) Assuming that the blood types for husband and wife are independent, $P(\text{Type B and Type A}) = (0.11)(0.40) = 0.044$.

(d) $P(\text{Type B and Type A}) + P(\text{Type A and Type B}) = (0.11)(0.40) + (0.40)(0.11) = 0.088$

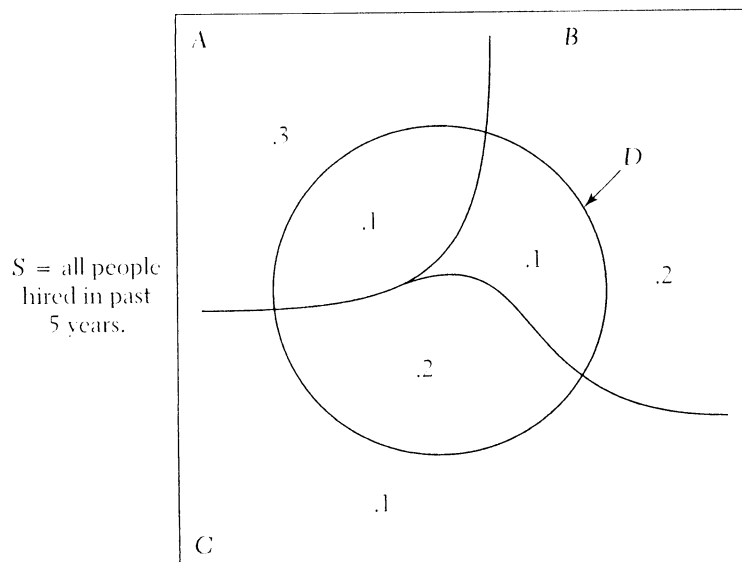
(e) $P(\text{Husband Type O or Wife Type O}) = P(\text{Husband Type O}) + P(\text{Wife Type O}) - P(\text{Husband and Wife Type O}) = 0.45 + 0.45 - (0.45)^2 = 0.6975$.

6.80 (a) $P(\text{both have Type O}) = P(\text{Amer. has O}) \cdot P(\text{Chin. has O}) = (.45)(.35) = .1575$.

(b) $P(\text{both have same Type}) = (.45)(.35) + (.4)(.27) + (.11)(.26) + (.04)(.12) = .2989$.

6.81 (a) To find $P(A \text{ or } C)$, we would need to know $P(A \text{ and } C)$. (b) To find $P(A \text{ and } C)$, we would need to know $P(A \text{ or } C)$.

6.82



$$P(O) = P(A \text{ and } D) + P(B \text{ and } D) + P(C \text{ and } D) = .1 + .1 + .2 = .4$$

6.83 (a) $P(\text{firearm}) = \frac{18,940}{31,510} = 0.6011.$

(b) $P(\text{firearm} | \text{female}) = \frac{2,559}{6,095} = 0.4199.$

(c) $P(\text{female and firearm}) = \frac{2,559}{31,510} = 0.0812.$

(d) $P(\text{firearm} | \text{male}) = \frac{16,381}{25,415} = 0.6445.$

(e) $P(\text{male} | \text{firearm}) = \frac{16,381}{18,940} = 0.8649.$

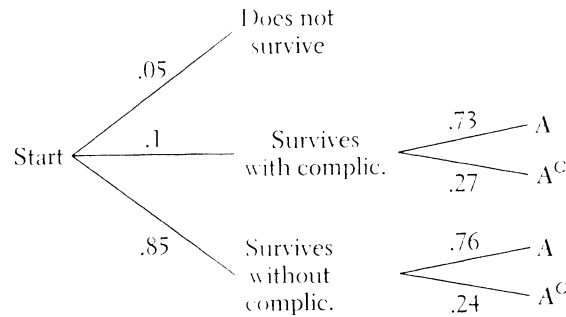
6.84 Let H = adult belongs to health club, and let G = adult goes to club at least twice a week.

$$P(G \text{ and } H) = P(H) \cdot P(G | H) = (.1)(.4) = .04.$$

6.85 $P(B | A) = P(\text{both tosses have the same outcome} | H \text{ on first toss}) = 1/2 = 0.5.$

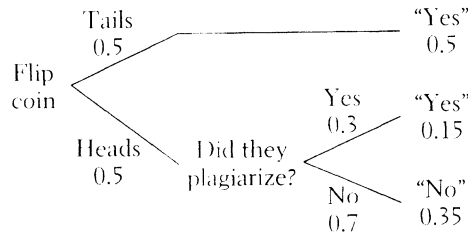
$P(B) = P(\text{both tosses have same outcome}) = 2/4 = 0.5.$ Since $P(B | A) = P(B)$, events A and B are independent.

6.86



$$P(A) = (.1)(.73) + (.85)(.76) = .719. \text{ Surgery gives John a slightly larger chance of achieving his goal.}$$

6.87 The response will be “no” with probability $0.35 = (0.5)(0.7)$. If the probability of plagiarism were 0.2, then $P(\text{student answers “no”}) = 0.4 = (0.05)(0.8)$. If 39% of students surveyed answered “no,” then we estimate that $2 \times 39\% = 78\%$ have *not* plagiarized, so about 22% have plagiarized.



Random Variables

7.1 (a) $P(\text{less than } 3) = P(1 \text{ or } 2) = \frac{2}{6} = \frac{1}{3}$. (b)-(c) Answers vary.

7.2 (a) BBB, BBG, BGB, GBB, GGB, GBC, BGG, GGG. Each has probability $1/8$. (b) Three of the eight arrangements have two (and only two) girls, so $P(X = 2) = 3/8 = 0.375$. (c) See table.

Value of X	0	1	2	3
Probability	$1/8$	$3/8$	$3/8$	$1/8$

7.3 (a) 1%. (b) All probabilities are between 0 and 1; the probabilities add to 1. (c) $P(X \leq 3) = 0.48 + 0.38 + 0.08 = 1 - 0.01 - 0.05 = 0.94$. (d) $P(X < 3) = 0.48 + 0.38 = 0.86$. (e) Write either $X \geq 4$ or $X > 3$. The probability is $0.05 + 0.01 = 0.06$. (f) Read two random digits from Table B. Here is the correspondence: 01 to 48 \Leftrightarrow Class 1, 49 to 86 \Leftrightarrow Class 2, 87 to 94 \Leftrightarrow Class 3, 95 to 99 \Leftrightarrow Class 4, and 00 \Leftrightarrow Class 5. Repeatedly generate 2 digit random numbers. The proportion of numbers in the range 01 to 94 will be an estimate of the required probability.

7.4

